2017 Spring PHYS2320 電磁學（Electromagnetism) Midterm (double sides)

[Griffiths Chs. 7.2-9] 2018/04/24, 10:10am – 12:00am, 教師：張存續 滿分 110

1. (20%) A capacitor C is charged up to a potential V and connected to an inductor L. At time $t = 0$ the switch S is closed.

(a) Find the current in the circuit as a function of time. (10%)

(b) How does your answer change if a resistor R is included in series with C and L. (10%)

2. (20%) Derive the Poynting theorem in **matter**, starting from the power delivered per unit volume $\frac{dW}{dt} = \int_V (\mathbf{E}\cdot\mathbf{J}_f)d\tau$.

(a) Show that the Poynting vector becomes $\mathbf{S} = \mathbf{E}\times\mathbf{H}$. (10%)

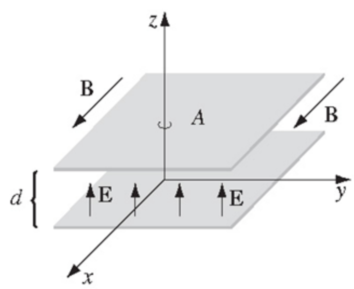
(b) Show that the energy density for linear media is $u_{\text{em}} = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D}+\mathbf{B}\cdot\mathbf{H})$. (10%)

3. (20%) A charged parallel-plate with uniform electric field ($\mathbf{E} = E\,\hat{\mathbf{z}}$) is placed in a uniform magnetic field ($\mathbf{B} = B\,\hat{\mathbf{x}}$) as shown in the figure below.

(a) Find the electromagnetic momentum in the space between the plates. (10%)

(b) Now a resistive wire is connected between the plates, along the z axis, so that the capacitor slowly discharges. The wire will experience a magnetic force. What is the total impulse delivered to the system, during the discharge? (10%)

[Hint: $\mathbf{F} = \oint_S \overleftrightarrow{\mathbf{T}}\cdot d\mathbf{a} - \varepsilon_0\mu_0\frac{d}{dt}\int_V \mathbf{S}d\tau$ and $T_{ij} \equiv \varepsilon_0(E_iE_j - \frac{1}{2}\delta_{ij}E^2) + \frac{1}{\mu_0}(B_iB_j - \frac{1}{2}\delta_{ij}B^2)$]



4. (25%) A hollow rectangular waveguide has a cross section of $a \times b = 2.54\ \text{mm} \times 1.27\ \text{mm}$.

(a) Estimate the cutoff frequencies for the first three modes (TE_{10}, TE_{20}, and TE_{01}). (10%)

(b) If the waveguide is 5.0 mm long and are closed at both ends (forming a resonant cavity), calculate the resonant frequency of the TE_{101} ($m = 1$, $n = 0$, and $l = 1$) mode. (10%)

(c) Qualitatively plot the dispersion relation ($\omega - k_z$ diagram) of the dominant TE_{10} mode. (5%)
[Hint: Detail how to calculate and express your answers in parts (a) and (b) in terms of GHz.]

5. (25%) A plane wave of frequency ω, traveling in the z direction with the electric field polarized in the x direction, is incident from vacuum (ε_0 and μ_0) to a dielectric with $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$. The incident wave is

$$\tilde{\mathbf{E}}_I(z,t) = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \text{ and } \tilde{\mathbf{B}}_I(z,t) = \frac{1}{c} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}}$$

where k is the corresponding wave number.

(a) Write down the reflected wave ($\tilde{\mathbf{E}}_R(z,t)$, $\tilde{\mathbf{B}}_R(z,t)$) and the transmitted wave ($\tilde{\mathbf{E}}_T(z,t)$, $\tilde{\mathbf{B}}_T(z,t)$). (10%)

(b) Express the reflected amplitude $\tilde{E}_{0R}$ and transmitted amplitude $\tilde{E}_{0T}$ in terms of the incident amplitude ($\tilde{E}_{0I}$). (10%)

(c) Find the reflection coefficient ($R \equiv I_R / I_I$) and the transmission coefficient ($T \equiv I_T / I_I$). (5%)
[Hint: $I \equiv \langle S \rangle = \frac{1}{2} v \varepsilon E_0^2$].